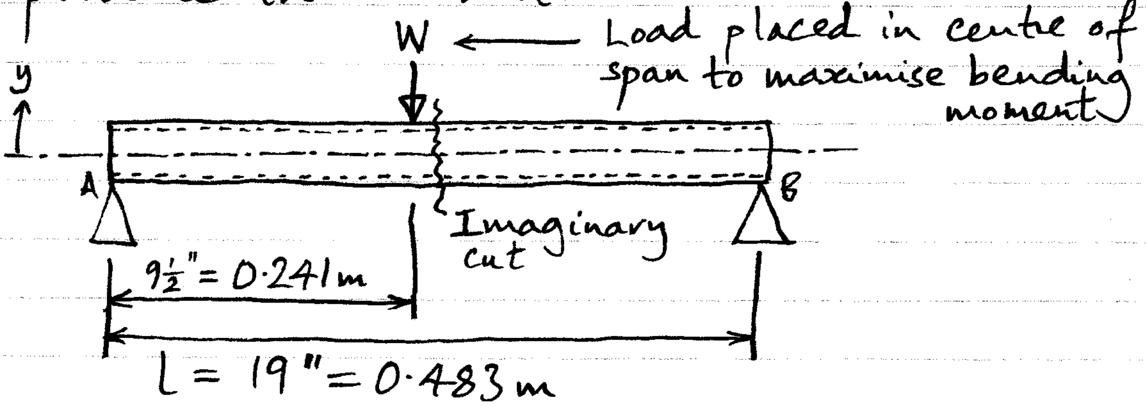
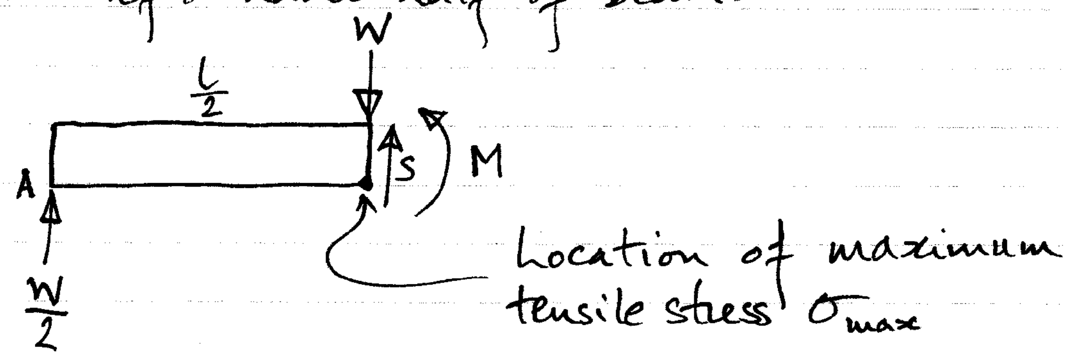
Make the following assumptions:

- Model the handrail as a simply-supported beam.
- Assume that the strength and stiffness of the vertical members and joints is not an issue.
- Assume that the load is a point load.
- Assume that localised buckling is not a problem.

These are reasonable assumptions when it comes to estimating the absolute maximum stress that might occur in the rail, but they are not reasonable assumptions for calculating the precise maximum stress that a given load will produce in the rail.



Consider left-hand half of beam:



Resolve forces to give shear force $S = \frac{W}{2}$.

Take moments about A to give:

$$\frac{WL}{2} - \frac{SL}{2} - M = 0$$

$$M = \frac{WL}{2} - \frac{W}{2}\cdot\frac{L}{2} = \frac{WL}{2} - \frac{WL}{4} = \frac{WL}{4}$$

This is the maximum bending moment anywhere in the beam.

Now use the standard formula $\sigma_{max} = -\frac{M_{max}\, y_{max}}{I}$.

Set σ_{max} equal to the yield stress σ_y.

$$\sigma_y = \frac{-M_{max}\, y_{max}}{I} = -\frac{WL}{4}\cdot\frac{y_{max}}{I}$$

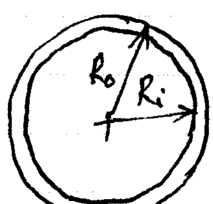
Rearrange to give $W = W_{max} = \frac{-4I\sigma_y}{y_{max}\, L}$

This is the maximum load that the rail can support at its centre without yielding, given the assumptions made at the start.

Assume $\sigma_y = 230\ N\,mm^{-2} = 230 \times 10^6\ N\,m^{-2}$

For $1\frac{1}{4}$" OD round tube with 0.065" wall:

$I = \frac{\pi}{4}\left(R_o^{\,4} - R_i^{\,4}\right)$ (standard formula)



$R_o = \frac{1.25}{2} \times 0.0254 = 0.0159\ m$

$R_i = R_o - 0.065 \times 0.0254 = 0.0142\ m$

Therefore $I = \frac{\pi}{4}(0.0159^4 - 0.0142^4)$

$$= 1.826 \times 10^{-8}\ m^4$$

$y_{max} = -R_o = -0.0159\ m$ (use $-R_o$ to give maximum tensile stress)

$$W_{max} = \frac{-4I\sigma_y}{y_{max}\,l}$$

$$= \frac{-4 \times 1.826 \times 10^{-8} \times 230 \times 10^6}{-0.0159 \times 0.483}$$

$W_{max} = 2187\ N = 223\ kgf = \underline{\underline{491\ lbf}}$

For $1\frac{1}{4}$" OD round tube with 0.120" wall:

$R_o = 0.0159\ m$ (same as before)
$R_i = R_o - 0.120 \times 0.0254 = 0.0129\ m$

Therefore $I = \frac{\pi}{4}(0.0159^4 - 0.0129^4)$

$$= 2.845 \times 10^{-8}\ m^4$$

$y_{max} = -0.0159\ m$ (same as before)

$$W_{max} = \frac{-4I\sigma_y}{y_{max}\,l}$$

$$= \frac{-4 \times 2.845 \times 10^{-8} \times 230 \times 10^6}{-0.0159 \times 0.483}$$

$W_{max} = 3408\ N = 347\ kgf = \underline{\underline{765\ lbf}}$

Christopher Tidy, 16 Jan '07.